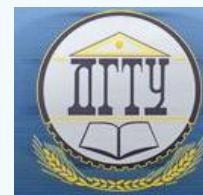# MACHINE BUILDING AND MACHINE SCIENCE

Statistical analysis of sizing features of dust generated under the mechanical metal-working

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Introduction. The paper considers the mathematical analysis of the fractional composition of dust generated during the operation of the rail-cutting machine. It is established that the studied polydisperse material is well described by the one-parameter exponential distribution. At the same time, the lognormal particle size distribution, whose parameters are determined by mathematical programming methods, seems adequate for the purposes of calculating cyclones. The work objective was to develop mathematical methods for correct averaging of the size and mass parameters of dust under the solid metal machining.

Materials and Methods. We studied the possibility of approximating experimental data by Rosin – Rammler distributions (classical, generalized three-parameter $P(x, D, n, m)$, and simplified exponential $P(x)$, in which $n = 1$). The corresponding results were compared to each other and to the data of approximation of the lognormal and double lognormal functions. These results indicate close approximation quality using the following model distributions $P(x)$:

- five-parameter double lognormal;
- three-parameter type of Rosin-Rammler;
- two-parameter classical Rosin - Rammler;
- one-parameter exponential.

Results. The primary physical analysis of cutting waste was carried out by the laboratory measuring complex *Fritsch Analysette* 22 *Compact* which uses the *LALLS – low angle laser scattering* method. The built-in software provides output of measurement results in primary graphic and digital forms. It was found that the simplest exponential distribution is best suited for a detailed analysis of the dust particle-size distribution based on the experimental data. This distribution enables reproduction of all the integral indicators provided by the instrumental measuring complex along with the graphical data.

Discussion and Conclusions. The results obtained can be used to rationalize the local suction machine, and the mathematical models and algorithms can be used for the parametric analysis of any dust captured by cyclones.

Keywords: metal cutting, dust, size distribution, statistics, mathematical programming

Introduction. Dust generated during machining (cutting, drilling, grinding, polishing, etc.) of hard metals and alloys can cause direct or indirect damage to human health and lead to environmental pollution. To eliminate these negative consequences, cyclones are widely used — devices of general and local air purification using aerodynamic capture of dust by inertia forces with its subsequent screening from the air stream into the accumulator [1]. The efficiency of cyclones is ensured by calculation, the purpose of which is to guarantee the capture of solid particles suspended in an air stream of a given category at minimal economic costs. Since particle capture is provided by competition between inertial and aerodynamic forces, the corresponding physical criteria form the basis for the calculation of cyclones [2]. The key components of these criteria are the dimensional and mass characteristics of dust particles. Due to the natural heterogeneity of industrial dust, these characteristics are of a statistical nature, which puts forward narrow requirements for the correctness of their averaging when calculating dust cleaning systems (cyclones). Thus, reliable scientific information on the dimensional and mass parameters of dust for various types of mechanical metalworking is a challenge since their practical application provides for a rational organization of dust collection through cyclones.

This study objective is to develop mathematical methods for the correct averaging of size and mass parameters of dust under the machining of hard metals. The results obtained were used to rationalize the local exhaust machine; and the mathematical models and algorithms developed by the authors are applicable without significant restrictions for the parametric analysis of any dust to be captured by cyclones.

The theoretical framework for the study. The dust particle is under the impact of gravity and inertial forces, which is proportional to the mass of the particle equal to $\rho \cdot x^3$, where ρ is the density of the forming material in [kg/m³]; x is the characteristic particle size in [μm]. Therefore, knowledge of the dust inertial properties comes down to knowledge of its density and characteristic size. The magnitude of the aerodynamic force acting on such a particle from the flow side is proportional to the square of its characteristic size x^2 and does not depend on density. The ratio of these forces appearing in the cyclone efficiency criterion is, respectively, proportional to the complex $\rho \cdot x$. However, two important circumstances should be taken into account: firstly, dust particles differ significantly in size and, secondly, the shape of each dust particle is unique and far from the standards used (sphere, cube, etc.). These features put forward very narrow requirements for the procedure of double averaging of the characteristic size of a dust particle by its component size and shape. Obviously, the technique averaging the characteristic size of dust particles is targeted: in our case, it corresponds to the calculation of cyclone efficiency.

For the first time, J. Sauter studied systematically the parametric averaging of polydisperse media [3, 4]. The key results of his work are as follows:

1. For various purposes, specifically averaged medium particle sizes from an inhomogeneous population are important. Since the average size is a value expressed in fractions of a meter, the following method of dimensional averaging of an ensemble of particles with a distribution function $F(x)$ and, accordingly, a probability density $P(x) = dF(x)/dx$ is obvious:

$$<x>_{ij} = [\int P(x)\, x^i\, dx / \int P(x)\, x^j\, dx]^{1/(i-j)} \; . \tag{1}$$

2. The formula (1) implies that all dust particles are characterized by a single size, i.e., they have the shape of a ball. Therefore, the value $<x>_{ij}$ is called the Sauter diameter and is designated as D_{ij} (most often, the Sauter diameter is understood as D_{32}).

3. If the shape of the particles is substantially irregular and is characterized by two or two parameters, the shape factor is also introduced into consideration. The physical meaning and practical application of various Sauter diameters are given in Table 1, and important information about the shape coefficient is in [5].

Numerous studies on the dispersion of various media and materials allow us to state [6] that the solid particles obtained as a result of single crushing are distributed in size according to the two-parameter Rosin – Rammler law:

$$F(x, D, n) = 1 - \mathrm{e}^{-(x/D)^n}, \qquad P(x, D, n) = 1/D \cdot (x/D)^{n-1} \cdot \mathrm{e}^{-(x/D)^n}, \qquad (2)$$

where the value $\langle x \rangle = D \cdot G(1 + 1/n)$ characterizes the average particle size, and n is the degree of dimensional heterogeneity of the ensemble (the smaller n, the higher the polydispersity of the powder).

Under multiple grinding, the powders consist of particles whose sizes satisfy the two-parameter lognormal Gauss-Kolmogorov distribution [6]:

$$P(x, D, \sigma) = \frac{\lg e}{\sqrt{2\pi} \cdot lg\sigma \cdot x} \cdot e^{-\frac{1}{2}\left(\frac{\lg x - \lg D}{\lg \sigma}\right)^2}. \qquad (3)$$

In the distribution (3), the parameter $\lg D$ corresponds to the conditional average particle size, and the parameter $\lg \sigma$ corresponds to the spread in real particle sizes around the conditional average.

The principal advantage of the Gauss – Kolmogorov model is the convenience of recalculating the values of D_{ij} from the linear Hatch – Choute relations [7], which links them to the quantities D and σ. The form of these relations is such that at any two known quantities D_{nm} and D_{kl}, all other quantities can be calculated.

It is important to note that the powder analyzed by the authors (rail cutting waste) is not necessarily described by the classical models given here. Firstly, the cutting process contains elements of both uniqueness (each contact of the abrasive wheel with the rail material is unique) and multiplicity (such interactions are extremely repetitive). Secondly, the shape of metallic filings is far from spherical. Finally, along with metal filings, cutting waste contains particles of a chipping abrasive. The content of the latter, due to the characteristics of the process and the requirements for the tool, can vary significantly. Thus, the study on the distribution of exhaust dust by particle size seems to be a practically important and scientifically significant task.

Experimental data. The initial physical analysis of the cutting waste was carried out using the *Fritsch Analysette 22 Compact* laboratory measuring complex through the *LALLS — low angle laser scattering* method [8]. The embedded software implements measurement results in primary graphic and digital forms. The disadvantages of the software part of the complex are: lack of detailed information on the conversion algorithms of the measured quantities and on the output data nature, as well as the quantitative errors of the data displayed in the form of graphs. In particular, it may seem that the scale of the differential distribution function (probability density) is given on printouts with a several-fold error. A deeper examination allows us to conclude that in fact on this graph, the dependence of the quantity $P(x_k)\, dx_k$ on x_k is shown, and the partition of particles by size into groups of width dx_k is not uniform. In addition, the integral characteristics of the particle D_{ij} size distribution calculated by the program are not documented, which requires their verification for compliance with both the original graphic data and the classical models of Rosin – Rammler and Gauss – Kolmogorov.

The noted circumstances, when evaluating the results of the analysis output by the built-in program, determine the implementation of additional measures: normalization of the initial curve of the differential distribution function, as well as checking the concordance of the integrated indices D_{ij} of both this distribution function and the basic model distributions. The implementation of these tasks requires high-quality digitization of the graphical results of fractional composition provided by the *Fritsch Analysette 22 Compact*, and this requires the specialized software [9, 10] and the development of appropriate data verification algorithms.

Digitization technique and experimental data verification. To digitize the initial graphic data of the dispersion analysis obtained using the *Fritsch Analysette 22 Compact* instrument, the specialized *Grafula* program [11] was used. This information freeware provides for translating graphically represented dependences into a tabular form.

The digitization procedure is reduced to reading the graph, placing the axes of the Cartesian coordinates on it, setting the scale and drawing a sufficient number of markers on the line representing the dependence.

The result of automatic digitization refers to the position of the points input by the user and is formed into an *Excel*-compatible table. The error in the resulting digital data has several components:

• error in the schedule generation due to the features of the *Fritsch Analysette 22 Compact* complex;

• defects in printing or displaying as an electronic photograph (screenshot);

• inaccuracy in digitizing the graph through the *Grafula* program;

• impossibility of accurately marking the curve on the graph due to limitations both in the resolution of the system and in the psychomotor capabilities of a person.

The last of these errors, apparently, is the most significant. The noted circumstances require supplemental checks of the digitization results to eliminate critical errors and evaluate correctly the result error.

To verify the digitization results, this study was carried out in several stages. At first, as applied to data in a normal particle size representation relative to those presented on a logarithmic scale, and then the initial analysis results were compared graphically. In this case, both digitized series were renormalized in order to eliminate the systematic error and ensure that the condition important for the distribution function $F(x)$ is fulfilled: $F(\infty) = 1$. Subsequently, on the basis of each of the results, the uniform model distributions were constructed whose empirical parameters were determined by mathematical programming methods [12] and compared. The last test stage was comparison of the averaged indicators of the dust dimensional composition calculated from the constructed model distributions with the integral characteristics of the distributions that are issued by the *Fritsch Analysette 22 Compact* instrument software.

It turned out that the distortion of the distribution function $F(x)$ as a result of the inability to extract it from the instrument directly in the digital representation is 15% when digitizing the graph in a linear (particle size) scale and 8% when digitizing the graph in a logarithmic scale. Fig. 1, from which it follows that both restored distributions practically coincide starting with a diameter of 10 μm, gives an idea of the coordination of the data obtained in two ways. At the same time, one and a half dozen of the first points available for reading from a graph having a logarithmic size scale indicate the presence of a significant proportion of small, most dangerous for humans particles smaller than 10 microns. This circumstance is secondary since active air purification systems capture predominantly small particles.

The fact, that at $x \geq 10$ μm, the graphic distributions displayed by the *Fritsch Analysette 22 Compact* at different scales coincide, is confirmed by the following test. If we approximate both series of experimental data by the Rosin – Rammler curve and compare the obtained curves, we can quantitatively evaluate the effect of the digitization error on the final result.
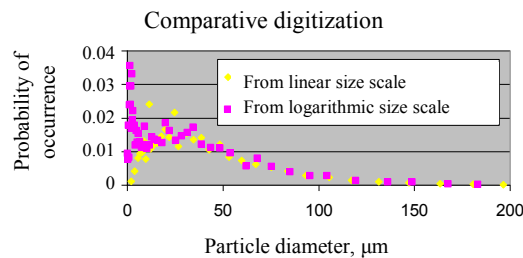

Fig. 1. Digitization results of dispersion analysis data of metal-abrasive dust

For approximation of the compared data presented in Fig. 1 by the model (2), the optimization problem related to mathematical programming was solved numerically [12]. The discrepancy between the actual and theoretical particle size distributions was minimized. If the discrepancy is determined by the Cartesian norm convenient for calculations, it is required to minimize the functional (Φ):

$$\Phi(x, D, n) = \sum_{k}[P(x_k, D, n) - P_k]^2 \rightarrow \min,$$

where k is the number of the nodal (tabular) point; P_k is the experimental probability density value; D and n are the described above parameters of the Rosin-Rammler model distribution.

Solving this problem through *Excel*, we obtain the following result. For the data obtained from a graph linear in x, the values of D and n are equal to 48.37 μm and 1.588 μm, respectively. Moreover, the curve approximating $P(x, D, n)$ is characterized by an average residual with experimental points 0 00274 and a correlation coefficient of 0 894. Similarly, solving the problem for 35 right points reconstructed from a logarithmic graph, we obtain the values of D = 48.61 μm, n = 1.346, residual ― 0.00187, correlation coefficient ― 0.934. The proximity of the parameters D and n for both tables, together with a small residual and a high correlation, indicate the approximate equivalence of the studied graphic images of the analyzed result. The correlation of the functions $P(x, D, n)$ with the calculated and above parameter values provides for the difference in the presentation of data in a formula form obtained from the compared graphic sources. Its value is 0.99.

Since the dust dimensional characteristics averaged on the basis of a certain realistic distribution are important, we calculate the set of indicators Dij according to the Rosin-Rammler model initialized above for the values of the parameters D and n obtained in different ways. Then, we compare the results to each other and to the D_{ij} integral indicators recorded by the *Fritsch Analysette 22 Compact*.

The purpose of comparing the analyzed graphic distributions to the integral indicators D_{ij} recorded by the device is to find out exactly which distribution is shown on these graphs. The fact that on the screen of the *Fritsch Analysette 22 Compact,* the D_{43} value coincides with the arithmetic mean diameter in five decimal places, and the D_{32} value coincides with the geometric mean size just as accurately, suggests that not the parameter $P(x_k)$, but the differential x^3-weighted distribution function is recorded:

$$dF(x_k) = P(x_k)\, x_k^3\, dx_k. \tag{4}$$

This is a key circumstance in interpreting the dimensional analysis results of dust particles based on histograms printed by the device. We verify the proposed hypothesis through calculating the moments D_{ij}. The results of comparing the calculations when accepting or rejecting the hypothesis of the authors to the corresponding numerical data recorded by the *Fritsch Analysette 22 Compact* are given in Table 1.

Table 1

Comparison of Sauter's dust diameters based on approximations of digitized graphic data

Diameter, μm	Hypothesis (4) is not true		Hypothesis (4) is true		Instrument numeric data, μm
	D = 48.4 μm, n = 1.588	D = 48.6 μm, n = 1.346	D = 48.4 μm, n = 1.588	D = 48.6 μm, n = 1.346	
D_{43}	90.2	107.1	43.4	44.4	43.5
D_{42}	83.2	97.9	30.4	27.5	20.9
D_{41}	75.2	87.1	15.6	12.5	8.18
D_{40}	65.5	73.6	7.61	6.11	4.23
D_{32}	76.7	89.5	21.3	17.0	10.0
D_{31}	68.6	78.5	9.38	6.62	3.55
D_{30}	58.9	64.9	4.26	3.15	1.95
D_{21}	61.4	68.8	4.13	2.57	1.25
D_{20}	51.6	55.2	1.90	1.35	0.66
D_{10}	43.4	44.4	0.88	0.71	0.59

As can be seen from the table, the hypothesis on the essence of the data obtained using the *Fritsch Analysette 22 Compact* service program is correct. This conclusion is crucial when interpreting the dust dimensional analysis results.

Statistical analysis results of graphic data and their interpretation. To better describe the finely dispersed part of the dust, we use the graphical data (Fig. 1) in a logarithmic size scale and approximate them by a five-parameter distribution corresponding to the weighted sum of two lognormal distributions:

$$P(x, D_1, D_2, \sigma_1, \sigma_2, \alpha) = \frac{\lg e}{\sqrt{2\pi} \cdot x} \cdot \left[\frac{\alpha}{\sigma_1} + \frac{1-\alpha}{\sigma_2} \right] \left[\alpha \cdot e^{-\frac{1}{2}\left(\frac{\lg x - \lg D_1}{\lg \sigma_1}\right)^2} + (1-\alpha) \cdot e^{-\frac{1}{2}\left(\frac{\lg x - \lg D_2}{\lg \sigma_2}\right)^2} \right],$$

where D_1 and D_2 are the peak position; σ_1, σ_2 are the peak width, α is the fraction of particles in the first mode.

Solving the corresponding optimization problem using *Excel* tools yields the following result: $D_1 = 45.2$ μm, $D_2 = 7.0$ μm, $\sigma_1 = 1.97$ μm, $\sigma_2 = 2.1$ μm, $\alpha = 0.858$. The average discrepancy between the approximating function and the initial data is 31%, and the correlation coefficient is 0.977. This is a satisfactory agreement given the high error of the experimental values (Fig. 1). Note that this result refers to the x^3-weighted true size distribution function of dust. It can be interpreted as follows: the bulk of the dust (about 85%) is made up of particles larger than 10 μm; therefore, for practical air purification, this distribution can be replaced by a two-parameter lognormal (2) one with the following parameters: $D = D_1 = 45.2$ μm and $\sigma_1 = \sigma_1 = 1.97$ μm. However, such a simplification will not allow a qualitative approximation of D_{ij} by $j < 3$, while the consideration of the fine fraction is significant for some applications, for example, for calculating all the used diameters D_{ij} and the moments of the distribution function $P(x)$. Below are the results of comparing the double lognormal and lognormal approximations of $P(x)$. Fig. 2 shows the experimental data approximation by a double lognormal distribution:

$$P(x) = 0.176 \cdot e^{-\frac{1}{2}\left(\frac{\lg x - 1.65}{0.416}\right)^2} + 0.563 \cdot e^{-\frac{1}{2}\left(\frac{\lg x - 0.898}{0.791}\right)^2}.$$

The mean-square ratio error is 0.31, and the correlation coefficient with experimental points is 0.977.
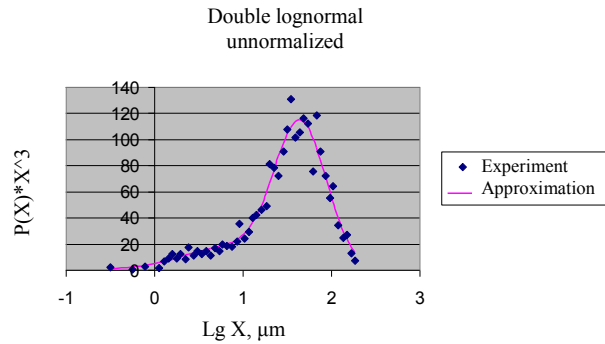

Fig. 2. Approximation of experimental data by double lognormal distribution

Fig. 3 shows the experimental data approximation by the lognormal distribution:

$$P(x) = 0.563 \cdot e^{-\frac{1}{2}\left(\frac{\lg x - 1.63}{0.435}\right)^2}.$$

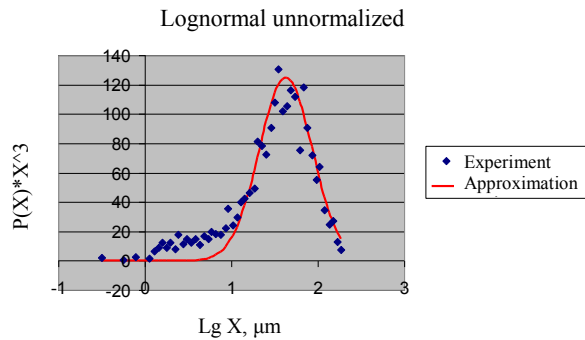The mean-square ratio error is 1.23, the correlation coefficient with experimental points is 0.970.

Fig. 3. Approximation of experimental data by lognormal distribution

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The possibility of approximating experimental data by the Rosin-Rammler distributions was also studied:

- by classic generalized three-parameter:

$$P(x, D, n, m) = (x/D)^m \cdot e^{-(x/D)^n} / \int (x/D)^m \cdot e^{-(x/D)^n} dx;$$

- by simplified exponential:

$$P(x) = 1/ D \cdot e^{-x/D}, \; n = 1.$$

Fig. 4 presents the experimental data approximation by the Rosin - Rammler distribution:

$$P(x) = 7.73 \cdot 10^{-3} \cdot x^{0.322} \cdot \exp[-(x/48.89)^{1.322}].$$

The mean-square ratio error is 0.47, and the correlation coefficient with experimental points is 0.972.

Fig. 5 shows the experimental data approximation by a three-parameter distribution of the Rosin-Rammler type:

$$P(x) = 6.66 \cdot 10^{-3} \cdot x^{0.4} \cdot \exp[-(x/66.18)^{1.235}].$$

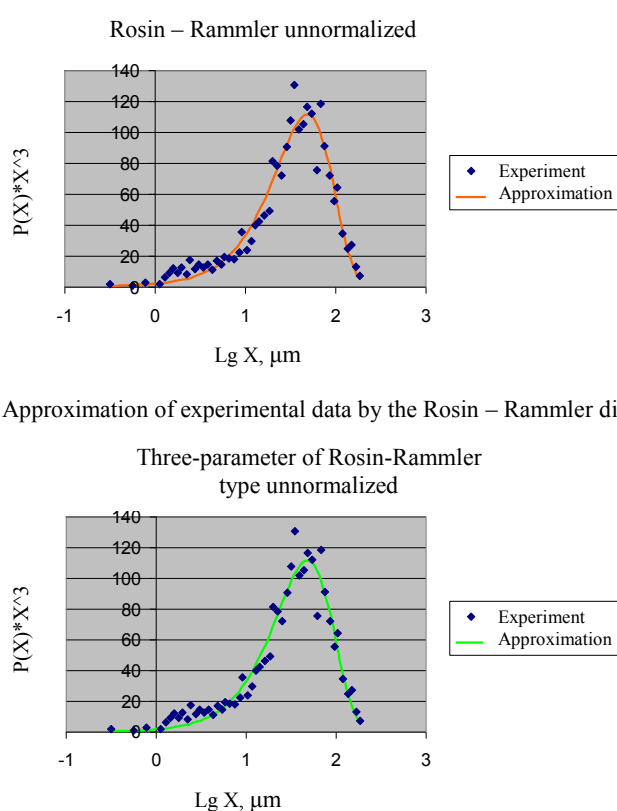The mean-square ratio error is 0.51, the correlation coefficient is 0.972.

Fig. 4. Approximation of experimental data by the Rosin – Rammler distribution
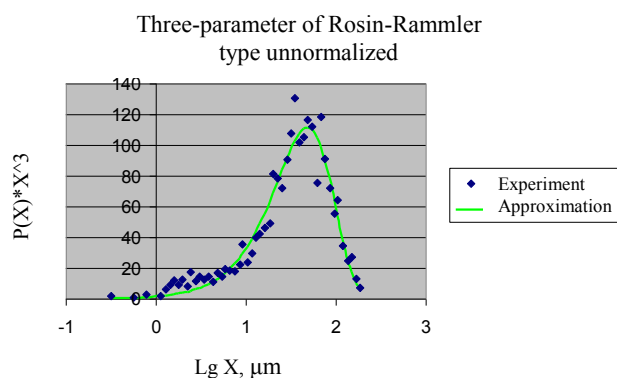

Fig. 5. Approximation of experimental data by three-parameter distribution of the Rosin-Rammler type

It follows from the analysis that for the problems other than cyclone calculations, it is convenient to approximate the experimental data of the authors by a one-parameter monotonically decreasing function: $P(x) = 1/D \cdot e^{x/D}$. Fig. 6 shows the approximation of experimental data by the exponential distribution $P(x) = 1/47.13 \cdot e^{x/47.13}$. The mean-square ratio error is 0.42, the correlation coefficient with experimental points is 0.948.
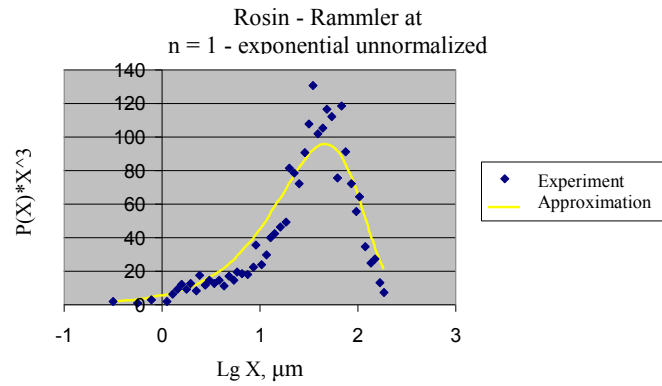
Fig. 6. Approximation of experimental data by exponential distribution

Table 2 shows the integrated indices of dust dispersion calculated on the basis of model distributions in comparison to the evaluated software of the *Fritsch Analysette 22 Compact*. Model distributions are initialized by graphic data.

Table 2

Integrated indices of dust dispersion

Parameter	Mathematical model used to evaluate					Actual value
	Lognormal	Double lognormal	Rosin - Rammler three-parameter	Rosin - Rammler	Exponential	
D_{43}, µm	51.7	44.8	44.7	45.2	43.6	43.5
D_{42}, µm	41.2	23.0	27.3	28.1	21.4	20.9
D_{41}, µm	32.0	9.45	12.2	12.9	8.61	8.18
D_{40}, µm	25.0	4.87	5.99	6.32	4.40	4.24
D_{32}, µm	32.8	11.8	16.6	17.5	10.3	10.0
D_{31}, µm	25.2	4.34	6.4	6.92	3.79	3.55
D_{30}, µm	19.6	2.32	3.07	3.28	2.03	1.95
D_{21}, µm	19.4	1.59	2.46	2.74	1.39	1.25
D_{20}, µm	15.1	1.03	1.32	1.42	0.904	0.858
D_{10}, µm	11.8	0.666	0.704	0.738	0.586	0.588
Mode, µm	52.1	45.2	48.9	48.4	43.9	43.4
MSR σ, µm	35.0	37.2	33.6	33.8	39.9	35.7
Asymmetry	1.49	1.17	1.19	1.20	1.41	1.25
Excess	2.28	1.49	1.46	1.50	1.66	1.66

The data in Fig. 4-6 and Table 2 provide for the comparison of the corresponding results to each other and to the approximation data of the lognormal and double lognormal functions. These results indicate a close approximation quality achieved using the five-parameter double lognormal, three-parameter Rosin-Rammler type, the two-parameter classical Rosin-Rammler type, and the one-parameter exponential model distributions $P(x)$. Meanwhile, the lognormal distribution does not correspond to the experimental data since it does not reflect the presence of a significant numerical fraction of very small particles in dust. At the same time, this distribution is best suited for the practical purpose of the study — optimization of the air purification system.

The approximation by one-parameter monotonically decreasing function $P(x) = 1/D \cdot e^{-x/D}$ demonstrates good agreement with the experiment for small-sized dust fractions, which, due to their representativeness, significantly affect the mean-square ratio approximation error.

Conclusion. The experimental data analysis performed by the authors regarding the size distribution of particles formed under a rail cutting reduces to the following:

1. The *Fritsch Analysette 22 Compact* is not optimal for studying the fractional composition of dust since it does not present measurement data in primary numerical form. The digitization of the graphs provided by this device is the major source of error in the interpretation of the relevant data.

2. For a detailed analysis of the dust particle size distribution based on the available experimental data (available from the authors), the simplest exponential particle size distribution $P(x) = 1/47.13 \cdot e^{-x/47.13}$ is best suited. Based on this distribution, all integrated indices provided by the instrumental measuring complex can be correctly reproduced, along with the graphical data.

3. The tasks of air purification require using a classic lognormal distribution:

$$P(x) = 0.563 \cdot e^{-\frac{1}{2}\left(\frac{\lg x - 1.63}{0.435}\right)^2},$$

whose parameters are calculated by the authors using mathematical programming methods. They made it possible to calculate the important for designing cyclones value $D_{32} = 32.8$ μm, which is three times higher than the numerical value produced by the *Fritsch Analysette 22 Compact*. This means that the application of the numerical values provided by the named device in the cyclone calculation will cause the inability to remove almost all large particles from the working area.

4. Due to the fact that the efficiency of the cyclone is affected not only by the value of D_{32}, but also by the density of the dispersed material consisting of steel and abrasive, a precise analysis is required to ensure the efficiency of air cleaning. Its subject is a separate study on the dispersion of metal and abrasive dust resulting from the operation of a rail-cutting machine.

References

1. Strauss W. Promyshlennaya ochistka gazov: per. s angl. [Industrial gas cleaning: transl. from English]. Moscow: Khimiya; 1981. 616 p. (In Russ.)

2. Ziganshin MG, Kolesnik AA, Posokhin VN. Proektirovanie apparatov pylegazoochistki [GTS design]. Moscow: Ehkopress - 3M; 1998. 505 p. (In Russ.)

3. Sauter J. Grössenbestimmung von Brennstoffteilchen. In: Forschungsarbeiten auf dem Gebiete des Ingenieurwesens. 1926. Heft 279.

4. Sauter J. Untersuchung der von Spritzvergasern gelieferten Zerstäubung. In: Forschungsarbeiten auf dem Gebiete des Ingenieurwesens. 1928. Heft 312.

5. Wadell H. Volume, Shape and Roundness of Quartz Particles. J. Geology. 1935;43(3):250-280.

6. Adushkin VV, Popel' SI, Shishaeva SI. Analiz melkodispersnoi fraktsii pri razrushenii gornykh porod vzryvom i obrazovanii skal'nykh opolznei [Analysis of the fine fraction under destruction of rocks by explosion and the formation of rock slips]. Journal of Mining Institute. 2007;171:32–38. (In Russ.)

7. Hatch J, Choate SP. Statistical description of the size properties of non-uniform particulate substances. J. Franklin Inst. 1929;207:369-387.

8. Kaye W, McDaniel JB. Low-Angle Laser Light Scattering – Rayleigh Factors and Depolarization Ratios. Applied Optics. 1974;13(8):1934-1937.

9. Bochkareva EA. Sravnitel'nyi analiz programm otsifrovki grafikov [Comparative analysis of the programs of digitizing graphs]. Modern Scientific Researchesand Inovations. 2015;11. URL: http://web.snauka.ru/issues/2015/11/60095 (accessed: 02.02.2020). (In Russ.)

10. Sharapova EhN, Dmitriev VL. Sistema otsifrovki graficheskikh dannykh [System of digitization of graphic data]. Software of Systems in the Industrial and Social Fields. 2011;1–2:166–171. (In Russ.)

11. Grafula — otsifrovka koordinat tochek otskanirovannykh grafikov dlya perenosa ikh v Excel: Matematicheskoe modelirovanie [Grafula - digitization of the coordinates of the points of scanned graphs for transferring them to Excel: Mathematical modeling]. URL : https://mmodelling.blogspot.com/2012/07/excel.html (accessed: 02.02.2020). (In Russ.)

12. Minoux M. Mathematical Programming: Theory and Algorithms. New York: John Wiley; 1986. 489 p.

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N. N. Azimova: analysis of subject literature, selection of research objective, task setting, planning and organization of the collaboration; experimental studies on dust size distribution by the LALLS method, selection and analysis of model distributions; quantitative assessment of the validity of the models used, their adjustment, analysis of the methodology applicability for objects similar to those studied, as well as the possibility to improve it regarding reliability and versatility; wording of the paper (30%). E. N. Ladosha: data digitization, parametric identification of the models based on graphical experimental distributions using statistical methods and *Excel* spreadsheets (15 %). S. N. Kholodova: examination of the technical experiment results (primary data on the dispersed composition of dust)